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FINITE ELEMENT ANALYSIS OF HEAT AND MASS TRANSFER THERMAL DIFFUSION EFFECT ON MHD FREE CONVECTION FLOW OF STRATIFIED VISCOUS FLUID

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ABSTRACT

The numerical solution of heat and mass transfer thermal diffusion effect on MHD free convection flow of stratified viscous fluid. Here we considered Visco-elastic, Darcy resistance terms, the constant permeability of the medium and neglected induced magnetic field in comparison to applied magnetic field. The velocity, temperature and concentration profiles are presented graphically and we obsereved that the temperature does not fluctuate with change in), K (Permeability parameter), Gr (Grash of Number), A (Thermal diffusion parameter), and Sc (Schmidth number). The concentration profile increases in increase in A But decreases with increase in Sc. Velocity profile rasises with increase in Gr, k, and A but it decreases with increase in M(Magnatic field).The equations are solved by Galerkin finite element method.

KEYWORDS: Heat and Mass Transfer, Free Convection, MHD, Porous Medium, Vertical Plate, Thermal Diffusion, Finite Element Method

INTRODUCTION

Heat transfer is that science with seeks to predict the energy transfer which may take place between material bodies as a result of a temperature difference. Thermodynamics teaches that this energy transfer is defined as heat. The science of heat transfer seeks not merely to explain how heat energy may be transferred, but also to predict the rate at which the exchange takes place under certain specified conditions. The fact a heat transfer rate is the desired objective of an analysis points out the difference between heat transfer and thermodynamics. Mass transfer is the net moment of mass from one location to another, mass transfer occurs in many process i.e., such as absorption, evaporation, drying, membrance filtration and distillation. Mass transfer is used by different scientific disciplines for different process and mechanisms. The phrase is commonly used in engineering for physical process that involves diffusive and convective transport of chemical species with in physical systems. In heat and mass transfer the transfer of heat is between a surface and moving fluid (liquid or gas), when they are at different temperature which is known as convection.

In geothermal reservoirs and geothermal we can find very important application of convection problem in a porous medium. And also we can observe the process of heat and mass transfer has application in chemical process industries, fluid fuel nuclear reactor, aeronautics and many engineering applications in which the fluid is working as medium. The large verity of technical and industrial applications has stimulated enthusiasm on the study of heat and mass transfer in the convection flows. Although the effect of stratification of the medium on the heat elimination process in a fluid is important, very little work has been recorded.

Ostrach[1953] studied free convective flow past a vertical plate. Even siegel [1958] investigated transient free convection from a vertical flat plate. Cheng and Lau [1977] and Cheng and Teckchandani [1977] obtained numerical solutions for the convective flow in a porous medium bounded by two isothermal parallel plates. However, porosity measurements by Benenati and Broselow's [1962] noticed that porosity is not constant varies from the surface of the plate to its interior, to which as a result permabulity also varies. Soundalgekar [1972] with regard to unsteady free convective flow stuided the effects of viscous dissipation on the flow past an infinite vertical porous plate. Chen *et al* [1980] have studied the combined effect of buoyancy forces from thermal and mass diffusion on forced convection. Bejan a and khair [1985] have investigated the mass transfer to natural convection boundary layer flow driven by heat transfer. The free convection on a horizontal plate in a saturated porous medium with prescribed heat transfer coefficient was studied by Ramanaiah and Malarvizhi [1991]. Lin h t and wu c m [1995] considered the problem of combined heat and mass transfer by laminar natural convection from a vertical plate. The mass transfer effects on free convection flow of an incompressible viscous dissipative fluid was studied by Manohard and Nagarajan [2001. Rushi Kumar b and Nagarajan [2007] have investigated mass transfer effects of MHD free convection flow of an incompressible viscous dissipative fluid past an infinite vertical plate.Recently, the effect of stratified viscous fluid on MHD free convection flow with heat and mass transfer past a vertical porous plate was discussed by Agrawal [2012].

In the present section we have considered the problem of Agrawal by introducing thermal diffusion and solved by Finite element method.

MATHEMATICAL ANALYSIS

The study of Heat and Mass Transfer Thermal Diffusion Effect on MHD Free Convection Flow of Stratified Viscous Fluid with assumptions of temperature of plate is constant, Viscous and Darcy's resistance terms are taken into account with constant permeability of the medium, The suction velocity normal to the plate is constant

$(V' = -U_0)$, Boussinesq's approximation is valid.

We considered a system of rectangular co-ordinates $O\ (x', y', z')$ where as $y' = 0$ on the plate and leading edge is z' axis. The density variation with temperature is considered while of all fluid properties remain constant. Due to the influence of density variation in the terms of the momentum and the energy equation and the variation of the expansion coefficient with temperature is neglected. The variations of density (ρ_0), viscosity (μ_0), elasticity (σ_0) and thermal conductivity (k_0) are considered as

$$\rho = \rho_0\, e^{-b'y'}, \mu = \mu_0\, e^{-b'y'}, \sigma = \sigma_0\, e^{-b'y'}, k_T = k_0\, e^{-b'y'}$$

where $\rho_{0\,,}\, \mu_0\,, \sigma_0\,, and\ k_0$ are the coefficients of density, viscosity, elasticity, and thermal conductivity respectively at $y' = 0, b' > 0$ represents the stratification factor.

Under these conditions, the problem is governed by the following system of Equations:

Equation of continuity: $\frac{\partial \mathrm{v}'}{\partial \mathrm{y}'} = 0$ (1)

Equation of Momentum:

$$\rho\left(\frac{\partial u'}{\partial t'} + V'\frac{\partial u'}{\partial y'}\right) = \frac{\partial}{\partial y'}\left(\mu\frac{\partial u'}{\partial y'}\right) - \left(\sigma B_0^2 + \frac{\mu}{K^1}\right)u^1 + \rho g\beta(T' - T'_\infty) + \rho g\beta^*(C' - C'_\infty) \quad (2)$$

Equation of Energy: $\frac{\partial T'}{\partial t'} + V' \frac{\partial T'}{\partial y'} = \frac{1}{\rho C_p} \frac{\partial}{\partial y'}\left(k_T \frac{\partial T'}{\partial y'}\right)$ (3)

Equation of Concentration: $\frac{\partial C'}{\partial t'} + V' \frac{\partial C'}{\partial y'} = D \frac{\partial^2 C'}{\partial y'^2} + D_1 \frac{\partial^2 T'}{\partial y'^2}$ (4)

Where, u', v' are the velocity components, T', C' are the temperature and concentration components, $\boldsymbol{v}$ is the kinematic viscosity. ρ is the density, σ is the electric conductivity, B_0 is the magnetic induction, k_T is the thermal conductivity and D is the concentration diffusivity, Cp is the specific heat at constant pressure, D1 is the thermal diffusivity.

The boundary conditions for the velocity, temperature and concentration fields are:

$$\begin{aligned} &u' = 0, T' = T_w,\ C' = C'_w \ at\ y' = 0 \\ &u' = 0, T' = T_\infty,\ C' = C_\infty, at\ y' \to \infty \end{aligned} \quad (5)$$

Introducing the non-dimensional variables

$(u, V) = \frac{(u', v')}{U_0}, \theta = \frac{(T' - T'_\infty)}{(T'_w - T'_\infty)}, C = \frac{(C' - C'_\infty)}{(C^1{}_w - C^1{}_\infty)}$,

$t = \frac{t' U_0^2}{v}, y = \frac{y' U_0}{v}, K = \frac{K_1' U_0^2}{v^2}$ is the permeability parameter, $U_0$is the reference velocity,

$M = \frac{\sigma B_0^2\, v}{\rho U_0^2}$ *is* Magnetic field parameter, $P_r = \frac{\mu\, C_p}{k}$ is Prandtl number,

$Sc = \frac{v}{D}$ is Schmidth number, $G_r = \frac{vg\beta(T_w - T_\infty)}{U_0^3}$ is Modified Grashof number for heat transfer,

$A = \frac{D_1(T_w - T_\infty)}{(C^1{}_w - C^1{}_\infty)}$is the thermal diffusion parameter , $N_0 = \frac{\beta^1}{\beta}\frac{(C^1{}_w - C^1{}_\infty)}{(T_w - T_\infty)}$ is the buoyancy ratio,

β is the thermal expansion coefficient, β^1 is the concentration expansion coefficient and, b is the stratification parameter. Other physical variables have their usual meaning.

Introducing the non-dimensional quantities describes above, the governing equations reduce to

$$\frac{\partial V}{\partial y} = 0 \quad (6)$$

$$\frac{\partial u}{\partial t} - (1 - b)\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} - \left(M + \frac{1}{K}\right)u + G_r(\theta + N_0 C) \quad (7)$$

$$\frac{\partial \theta}{\partial t} - (1 - \frac{b}{P_r})\frac{\partial \theta}{\partial y} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial y^2} \quad (8)$$

$$\frac{\partial C}{\partial t} - \frac{\partial C}{\partial y} = \frac{1}{S_c}\frac{\partial^2 C}{\partial y^2} + A\frac{\partial^2 \theta}{\partial y^2} \quad (9)$$

and the corresponding boundary conditions are:

$$\left.\begin{aligned} &u = 0, \theta = 1, C = 1\ at\ y = 0 \\ &u = 0, \theta = 0, C = 0\ as\ y \to \infty \end{aligned}\right\} \quad (10)$$

METHOD OF SOLUTION

By applying the Galerkin finite element method for equation (7) over a typical two-nodded linear

element (e) ($y_j \leq y \leq y_k$) is

$$\int_{y_j}^{y_k} N^T \left[\frac{\partial u}{\partial t} - (1-b)\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} - \left(\mathrm{M} + \frac{1}{\mathrm{K}}\right)\mathrm{u} + G_r(\theta + N_0 C)\right] dy \tag{11}$$

$$\int_{y_j}^{y_k} \left[\frac{\partial N}{\partial y}.\frac{\partial u^{(e)}}{\partial y} + N^T \left((1-b)\frac{\partial u^{(e)}}{\partial y} - \frac{\partial u^{(e)}}{\partial t} - \left(\mathrm{M} + \frac{1}{\mathrm{K}}\right) u^{(e)} + R^* \right)\right] dy = 0 \tag{12}$$

$$where\ R^* = G_r(\theta + N_c C), N = [N_j, N_k], \emptyset^{(e)} = \begin{bmatrix} u_j \\ u_k \end{bmatrix},$$

$$u^{(e)} = N.\emptyset^{(e)},\ N_j = \frac{y_k - y}{l^{(e)}}, N_k = \frac{y - y_j}{l^{(e)}},\ l^{(e)} = y_k - y_j = h,$$

The element equation given by

$$\int_{y_j}^{y_k} \begin{bmatrix} N_j^{'}N_j^{'} & N_j^{'}N_k^{'} \\ N_k^{'}N_j^{'} & N_k^{'}N_k^{'} \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} dy - (1-b) \begin{bmatrix} N_jN_j^{'} & N_jN_k^{'} \\ N_kN_j^{'} & N_kN_k^{'} \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} dy + \begin{bmatrix} N_jN_j & N_jN_k \\ N_kN_j & N_kN_k \end{bmatrix} \begin{bmatrix} \dot{u}_j \\ \dot{u}_k \end{bmatrix} dy +$$

$$\left(\mathrm{M} + \frac{1}{\mathrm{K}}\right) \begin{bmatrix} N_jN_j & N_jN_k \\ N_kN_j & N_kN_k \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} dy - R^* \begin{bmatrix} N_j \\ N_k \end{bmatrix} dy = 0 \tag{13}$$

Now put row corresponding to the node i to zero, from equation (13) the difference schemes with $l^{(e)} = h$ is

$$\frac{h}{6}(\dot{u}_{i-1} + 4\dot{u}_i + \dot{u}_{i+1}) + \left(\left(\mathrm{M} + \frac{1}{\mathrm{K}}\right)\frac{h}{6} - \frac{(1-b)}{2} - \frac{1}{h}\right)u_{i-1} + \left(\frac{2}{h} + \left(\mathrm{M} + \frac{1}{\mathrm{K}}\right)\frac{h}{3}\right)u_i + \left(\left(\mathrm{M} + \frac{1}{\mathrm{K}}\right)\frac{h}{6} + \frac{(1-b)}{2} - \frac{1}{h}\right)u_{i+1} =$$

$$\mathrm{R\ and\ here\ R} = R^* \frac{l^{(e)}}{2} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \tag{14}$$

Using the Cranck-Nicolson method to the equation (14), we obtain:

$$A_1 u_{i-1}^{j+1} + A_2 u_i^{j+1} + A_3 u_{i+1}^{j+1} + A_4 u_{i-1}^{j} + A_5 u_i^{j} + A_6 u_{i+1}^{j} = R \tag{15}$$

$$B_1 \theta_{i-1}^{j+1} + B_2 \theta_i^{j+1} + B_3 \theta_{i+1}^{j+1} + B_4 \theta_{i-1}^{j} + B_5 \theta_i^{j} + B_6 \theta_{i+1}^{j} = 0 \tag{16}$$

$$C_1 c_{i-1}^{j+1} + C_2 c_i^{j+1} + C_3 c_{i+1}^{j+1} + C_4 c_{i-1}^{j} + C_5 c_i^{j} + C_6 c_{i+1}^{j} = R_1 \tag{17}$$

$$A_1 = \left(1 - 6r + 3p(1-b) + \left(\mathrm{M} + \frac{1}{\mathrm{K}}\right)k\right),\ A_2 = \left(4 + 12r + 4\left(\mathrm{M} + \frac{1}{\mathrm{K}}\right)k\right), A_3 = \left(1 - 6r - 3p(1-b) + \left(\mathrm{M} + \frac{1}{\mathrm{K}}\right)k\right),$$

$$A_4 = \left(1 + 6r - 3p(1-b) - \left(\mathrm{M} + \frac{1}{\mathrm{K}}\right)k\right),\ A_5 = \left(4 - 12r - 4\left(\mathrm{M} + \frac{1}{\mathrm{K}}\right)k\right), A_6 = \left(1 + 6r + 3p(1-b) + \left(\mathrm{M} + \frac{1}{\mathrm{K}}\right)k\right)$$

$$B_1 = \left(1 - 6r\frac{1}{P_r} + 3p(1-b)\right),\ B_2 = \left(4 + 12r\frac{1}{P_r}\right), B_3 = \left(1 - 6r\frac{1}{P_r} - 3p(1-b)\right),$$

$$B_4 = \left(1 + 6r\frac{1}{P_r} - 3p(1-b)\right), B_5 = \left(4 - 12r\frac{1}{P_r}\right),\ B_6 = \left(1 + 6r\frac{1}{P_r} + 3p(1-b)\right),$$

$$C_1 = \left(1 - 6r\frac{1}{S_c} + 3p\right),\ C_2 = \left(4 + 12r\frac{1}{S_c}\right), C_3 = \left(1 - 6r\frac{1}{S_c} - 3p\right),$$

$$C_4 = \left(1 + 6r\frac{1}{S_c} - 3p\right), C_5 = \left(4 - 12r\frac{1}{S_c}\right), C_6 = \left(1 + 6r\frac{1}{S_c} + 3p\right),$$

$$R_1 = \left(\frac{\theta_{i+1} - 2\,\theta_i + \theta_{i-1}}{h^2}\right)$$

Here where h=0.1 and k=0.05 mesh sizes along y direction and time direction considered for computations.

$$A_i X_i = B_i, i = 1,2,3 \ldots. \quad (18)$$

Where A_i's are matrices of order n and X_i *and* B_i are column matrices having n-components.

RESULTS AND DISCUSSIONS

Velocity distribution of fluid flow is plotted and shown in figure 1 for n=0.1, Pr=0.71, No=1.5, t=0.1, b=0.1 for with different value of M, Gr,A ,K, and Sc. figure 1 shows that all velocity graphs are increased sharply for y=1.2 as such velocity in each graph begins to decrease and tends to zero with the increasing in y. It is also observed from figure 1 the velocity decreases with increase in M but it increases with the increase in Gr, A, K and Sc. But the temperature remain constant parameters taken for velocity. The concentration profiles for different values of diffusion parameter *A* are shown in figure 2. It is noticed that concentration profiles increases with the increase of *A* the concentration profiles are higher when *Sc* is 0.4, compared with *Sc* is 2. It is also observed that concentration increases with the increase in A, but it decrease with the increase in Sc in figure 2. The graph of *Sc* verses the *C* presented in figure 3. The *Sc* effect is leads to decreases the concentration profiles are observed form the figure.

CONCLUSIONS

- The velocity increases with the increase in A (Thermal diffusion parameter).
- The concentration also increases with the increase in A.

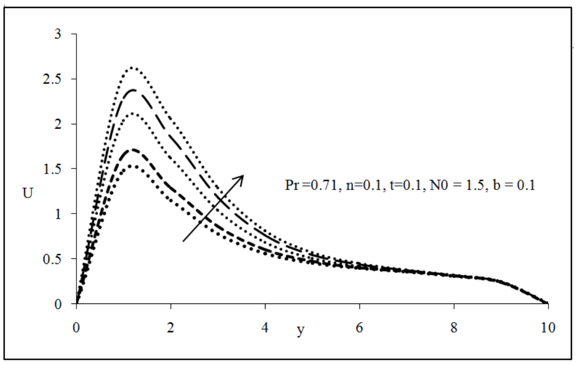

Figure 1: Velocity Profiles

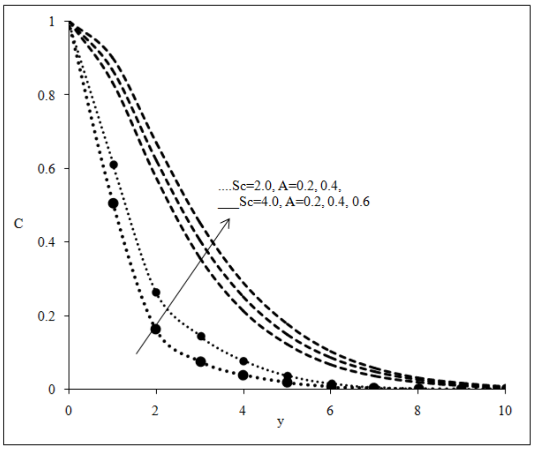

Figure 2: Concentration Profiles

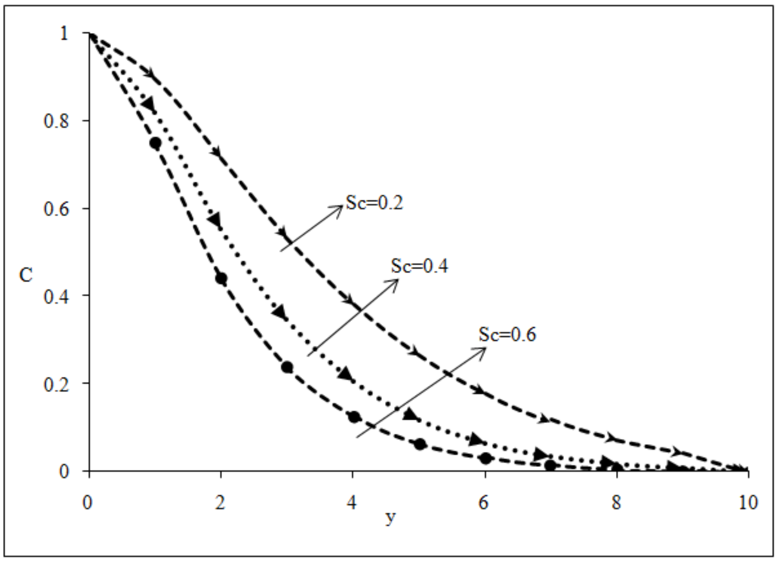

Figure 3: Concentration Profiles for Different Values of Sc

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